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PRELIMINARY NOTES ON THE PRESENCE AND THE INFLUENCE OF THE VERNACULAR IN AN ANONYMOUS HEBREW ARITHMETICAL TREATISE FROM FIFTEENTH-CENTURY CENTO (VERONA, BIBLIOTECA CIVICA, MS. 33)[1]

Introduction

The kernel of this article is the hitherto-unpublished Hebrew arithmetical treatise, influenced by the vernacular, found in Ms. 33 at the Civic Library of Verona,[2] a *unicum* which drew my attention during my investigation of the presence and the role of the vernacular in Hebrew mathematical treatises in Renaissance Italy. Neither the name of the author nor the place of composition are given. The colophon reveals that the codex was produced in Cento in 1460/1. Whether the scribe, who also copied two other non-mathematical treatises by named authors, is also the author of the arithmetical treatise would be no more than a speculation.[3]

Before discussing Ms. 33 and several interesting cases of vernacular presence and influence in its arithmetical part, the introductory parts of the article succinctly present three themes a priori relevant for the contextual study. The first theme concerns Jewish history and intellectual activities in Cento, where the codex was copied. The second field is Judeo-Italian, its definition and its presence in various literary genres during the Renaissance. This linguistic facet is vital to our discussion given that the vernacular in Hebrew letters is explicitly and implicitly discernible in the arithmetical treatise. The third area relevant to our study is the tradition of *abbaco* in Renaissance Italy. Although I have not been able so far to find a specific *abbaco* source directly connected to our Hebrew treatise, the palpable influence of the vernacular, some examples of which will be shown in this article, seems to suggest that the author, apart from his apparent knowledge of medieval Hebrew arithmetic, may have also been educated either by *abbaco* teachers or books. In any case, the author intended the book to be a proper textbook, given its structure and pedagogical formulations.

Jewish presence and intellectual activities in Cento

The most ancient surviving document proving the existence of Jews in Cento dates to 1390 and relates to the banker Manuele di Gaudio di Roma.[4] The Jews of Cento often originated from Bologna, some of whom were involved in moneylending and banking, in Cento and beyond. Lending money from Jews seems to have helped resolve the scarce disposability of cash in the area. Between 1390 and the end of the fifteenth century, Jewish presence in Cento is well documented, albeit in a non-systematic and incomplete manner. From 1449, Cento was under Bolognese episcopal domination. In 1455, Pope Callixtus III issued a bull, granting the Jews there all the exemptions given to the non-Jewish inhabitants of Cento and Pieve di Cento except

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[2] Also catalogued as Ms. 83.1.

[3] See footnote 18.

[4] For details on Manuele's intricate life-path and financial activities, see M.G. Muzzarelli, *Ebrei a Cento in epoca medioevale*, in AA. VV., *Gli ebrei a Cento e Pieve di Cento fra Medioevo ed Età moderna (atti del convegno di studi storici tenutosi a Cento il 22 aprile 1993)*, Cento 1994, pp. 13-22.

for pacts made between the Jews and the town communities. The bull also included exemption from taxes for Jews, which was not the case for other residents of Bologna and the surrounding countryside.[5]

Regarding known scientific and intellectual activities of Jews in Cento, there is a notary document dating to the 1460s, in which the physician Moses Ashkenazi is mentioned, who treated the benumbed leg of a Christian child.[6] Manuscripts copied in Cento or related to it (e.g. in which Cento is mentioned) exist from the end of the fourteenth century, as well as fragments which ended up as covers to register books, known as the *Italian Genizah*. The contents of these manuscripts and fragments span a wide variety of subjects: Bible, Biblical commentaries, Halakha, Kabbalah, philosophy, and grammar.[7] It is noteworthy to mention a Siddur from 1499, copied in Cento by David ben Menachem de Arli, in the vernacular, which was commissioned for Bona (Tovah) Bonaventura.[8] The codex at the centre of this article reveals interest in poetry, satire and arithmetic.

Judeo-Italian

The term *Judeo-Italian* in our mid-fifteenth-century context refers to varieties of the Italian vernacular written in Hebrew characters. Judeo-Italian is attested in different literary genres: Biblical, Talmudic and Halakhic texts and their commentaries, prayer books, dictionaries, grammar books, poetic works as well as scientific treatises. Judeo-Italian is either the main language of the text, as is the case of translations and original works, or it can be found alongside Hebrew (and at times, other languages, such as Judeo-Arabic) in glossaries. Furthermore, Judeo-Italian often served as an ancillary tongue present in interlinear or marginal comments and glosses. It enabled access to Jewish literature to those Jews for whom Hebrew was less comprehensible than the vernacular (e.g., women), but who were still able to read the Hebrew script.[9]

[5] *Ivi*, pp. 13-15, 22.

[6] *Ivi*, p. 22.

[7] M. Perani, *Manoscritti e frammenti ebraici copiati o conservati a Cento e Pieve di Cento*, in AA. VV., *Gli ebrei a Cento e Pieve di Cento fra Medioevo ed Età moderna (atti del convegno di studi storici tenutosi a Cento il 22 aprile 1993)*, Cento 1994, pp. 93-156.

[8] S. Campanini, *Il formulario (Siddur) di preghiere in volgare copiato a Cento nel 1499*, in AA. VV., *Gli ebrei a Cento e Pieve di Cento fra Medioevo ed Età moderna (atti del convegno di studi storici tenutosi a Cento il 22 aprile 1993)* 1994, Cento 1994, pp. 157-189.We also know of a scribe by the name of Isaac ben Joel of Cento who copied a Maḥzor of Roman rite in 1397, see footnote 16.

[9] For further details on the presence of Judeo-Italian in other works, as well as an elaboration of the nuanced definition of Judeo-Italian as a social dialect and the analysis of its role and evolution throughout the centuries, see S. Debenedetti Stow, *Il Giudeo-Italiano dal Medioevo al primo Rinascimento*, in «La RassegnaMensile di Israel» Vol. 82, No. 2-3 (2016), pp. 257-284, as well as L. Cuomo, *Il giudeo italiano e le vicende linguistiche degli ebrei d'Italia*, in *Italia Judaica, Atti del I Convegno Internationale (Bari 18-22 Maggio 1981)*, Roma 1983, pp. 427-454, M.L. Modena, *Il giudeo-italiano: riflessioni sulle fonti*, in «Materia Giudaica»VIII/1 (2003), pp 65-73, S. Natale, *L'elegia giudeo-italiana, Edizione critica e commentata*, Testi e culture in Europa 26, Pisa 2018, and A.D. Rubin, *Judeo-Italian*, in L. Kahn, A.D. Rubin (eds.), *Handbook of Jewish Languages*, Brill, Leiden-Boston 2015, pp. 298-364. Out of the large body of treatises written in Judeo-Italian I shall mention here the thirteenth-century Judeo-Italian glossary of scientific and philosophical Hebrew terms in Maimonides's *Guide of the Perplexed* composed by Moses da Salerno, see G. Sermoneta, *Un glossario filosofico ebraico-italiano del XIII secolo*, Ateneo, Roma 1969, as well as the version of *Song of Songs* dating to the fifteenth century and deriving from central-northern Italy, see Id., *Un volgarizzamento giudeo-italiano del Cantico dei Cantici*, Sansoni, Florence 1974. The Siddur in the vernacular copied in 1499 in Cento, mentioned above, is another interesting component of Judeo-Italian literature. For the presence of Judeo-Italian in the transmission of Hebrew arithmetic into Italy in the thirteenth century, see I. Wartenberg, *A note on Judeo-Italian arithmetical terminology in the transmission of Abraham Ibn 'Ezrā's Sefer ha-Mispar*, in «Materia Giudaica» XXV (2020), pp. 289-283.

Abbaco

Within the vast field of Renaissance mathematics, I wish to linger on one subject which may be directly or indirectly related to the Hebrew arithmetical text at the centre of this article, *abbaco*.[10] *Abbaco* is a branch of practical mathematics, which became very popular in the larger commercial development of the Italian Peninsula, especially in cities such as Venice, which was the centre of international trade and banking. The economic growth during the Italian Renaissance created the need for knowledge of mercantile skills, at the basis of which lies sophisticated applied mathematics. The famous painter Piero della Francesca (ca. 1415-1492), who authored an *abbaco* text, gives the following incipit: 'Being invited to write down something on abaco, which for merchants is paramount [...].'[11] This type of applied mathematics was remote from the medieval as well as ancient philosophical study of numbers that was often applied to metaphysics and theology.

The term *abbaco* derives from the title of the treatise *Liber Abaci* by Leonardo Pisano, also known as Fibonacci, which was composed in 1202. The author created a revised edition in 1228, which he dedicated to Michael Scot.[12] Abbreviated vernacular versions of the *Liber Abaci* appeared in Italy towards the end of the thirteenth century. Numerous other *abbaco* treatises were composed during the Renaissance.[13] *Liber Abaci* was extremely influential in spreading the learning of the Hindu-Arabic numeral system within Christian Europe, a system which gradually replaced the cumbersome Roman system which had been dominating the European medieval mathematical literature and practice. It may seem curious, however, that Fibonacci chose the word *abacus* to appear in the title of his work, not only because his treatise did not teach how to use the abacus, but in fact, the decimal place-value system at the heart of his teaching rendered the use of the abacus completely unnecessary.[14]

Among the common arithmetical subjects found in *abbaco* books are the place-value decimal system, the four basic arithmetical oper-

[10] The spelling *abbaco* is found in vernacular sources alongside *abaco* and *abbecho*, and even *abicho*, see A. Petrocchi, *Mercantile Arithmetic in Renaissance Italy: A Translation and Study of Selected Passages from a Vernacular Abbaco Work*, in «Ceræ: An Australasian Journal of Medieval and Early Modern Studies» 3 (2016), pp. 2, 8, 25. I have chosen to use the most prevalent spelling used by historians of mathematics.

[11] See *ivi*, pp. 4, 26.

[12] Boncompagni published the first modern edition of Leonardo's *Liber Abaci*, see B. Boncompagni, *Scritti di Leonardo Pisano, matematico del secolo decimoterzo: Il liber abbaci di Leonardo Pisano pubblicato secondo la lezione del codice magliabechiano C. 1, 2616, Badia Fiorentina, no. 73*, Tipografia delle Scienze Matematiche e Fisiche, Rome 1857. A new edition is being prepared at the University of Naples Federico II by a multidisciplinary research team, led by Giuseppe Germano, see G. Germano, *New Editorial Perspectives on Fibonacci's Liber Abaci*, in «Reti Medievali Rivista» 14(2) (2013), pp. 157-173 and C. Carotenuto, *Observations on selected variants of Fibonacci's Liber Abaci*, in «Reti Medievali Rivista»14(2) (2013), pp. 175-188. A first translation into a modern language, English, was made by Sigler, see L. Sigler, *Fibonacci's Liber Abaci, A Translation into Modern English of Leonardo Pisano's Book of Calculation*, Springer, New York, Berlin, Heidelberg 2002.

[13] For example, *Arte Giamata Aresmetica*, see Petrocchi, *Mercantile Arithmetic*, cit., pp. 1-30.

[14] A detailed analysis of *Liber Abaci* is beyond the scope of this article. I shall only mention briefly that Høyrup has argued against the common view that Fibonacci was the founding father of the *abbaco* culture. He claimed that, in fact, no simple matters were borrowed from *Liber Abaci* into later *abbaco* books, only sophisticated subjects, which were not understood by the compilers, and which served a mere ornamental function and a way in which the redactors could show off in front of others, who understood no more. Høyrup claimed that Fibonacci should be regarded as an extraordinary representative of the *abbaco* culture, who had grown taller and more conspicuous than others, becoming a culture hero of the *abbaco* culture. See J. Høyrup, *Leonardo Fibonacci and Abbaco culture. A proposal to invert the roles*, in «Revue d'histoire des mathématiques» (2005), pp. 23-36. Høyrup's thesis has been contested by other historians of mathematics, such as Van Egmond and Oaks. On the heated debate regarding the contribution of Fibonacci to the *abbaco* culture, see E. Caianiello, *Les sources des textes d'abaque italiens du XIVe siècle*, in «Reti medievali Rivista» 14(2) (2013), pp. 189-210.

ations, including the multiplication table, as well as numerous algorithms for multiplication of integers and fractions. Some *abbaco* books included geometry, algebra, calendars, and astrology. However, the kernel of *abbaco* study is the solution of practical arithmetical problems. These problems mainly involve business transactions, such as barter, currency exchange and calculation of interest, but also occasional recreational puzzles. The authors generally used a conversational style, with no attempt to generalize or theorize, and each problem was solved individually. *Abbaco* required much practical ingenuity as well as memorization. Its study was integrated into vernacular schools, which also taught reading and writing and at times, bookkeeping. Learning to read and to write was done directly in the vernacular, with only minimal instruction of Latin, the latter serving as a notarial tool. *Abbaco* schools were spreading across northern Italy between the fourteenth and the sixteenth century. Up to the sixteenth century, the majority of *abbaco* teachers and authors came from Florence. A famous graduate is Niccolò Machiavelli (1469-1527), who attended a vernacular school in his natal town, Florence, from the age of almost 11, for almost two years. In general, there was a separation between Latin and vernacular cultures. The scholastic curriculum and the teachings in *scuole d'abbaco* were usually not conceived as complementary study programs but rather, alternative, and in many ways, parallel. Furthermore, attending a Latin school was often deemed a luxury.

The study of *abbaco* (or even mathematics in general) was not usually part of the curriculum of Latin schools. Although some humanists recognized the importance of mathematics, and many were quite knowledgeable in the field, they seldom taught the subject, and even when they did, they either used classical texts (in line with the humanistic inclination to revert to ancient texts, for example from the mathematical-philosophical legacy of Boethius), or other books pertaining to medieval scholastic mathematics. Yet, although *abbaco* was often regarded by humanists as suitable for artisans but unbefitting for the social status and career goals of Latin students, there were other voices, too. For example, the humanist Leon Battista Alberti, the epitome of a fifteenth-century Renaissance man, recommended in his *Della famiglia* that children learn *abbaco* (as well as geometry) for pleasure and utility.[15]

Verona, Biblioteca Civica, Ms. 33

The Hebrew treatises in this paper codex were copied by a single hand in brown ink in semi-cursive Italian script deriving from Central-Northern Italy, typical of the second half of the fifteenth century. The first folios are damaged due to humidity. There are no indications of the scribe's identity, neither through acrostics nor explicitly.[16] As for its contents, the codex includes the following three treatises: (i) Fols. 1r-7r: אמרי נתנאל בן שלמה (The sayings of Netan'el

[15] For further details on *abbaco* teaching and the comparison between the vernacular and Latin school systems, see P.F. Grendler, *Schooling in Renaissance Italy. Literacy and Learning, 1300-1600*, J. Hopkins University Press, Baltimore-London 1989, pp. 306-319 and P. Lucchi, *Leggere, scrivere e abbaco : l'istruzione elementare agli inizi dell'età moderna*, in *Scienza, credenze occulte, livelli di cultura*, Firenze 1982, pp. 101-119.

[16] The film number of the codex at the Institute of Microfilmed Hebrew Manuscripts (IMHM) at the National Library of Israel is F 32674. Elementary details about the codex can be found in G. Tamani, *Manoscritti ebraici nella Biblioteca Communale di Verona*, in «Rivista degli Studi Orientali», Vol. 45 (3/4) (1970), pp. 235. Tamani does not mention the scribe's name in his catalogue, and indeed, no name is to be found in the codex. However, more than two decades later, Tamani mentions Isaac ben Joel of Cento as the copyist of *'Even Boḥan*. If this were the case, then Isaac would be the scribe of all three treatises in the codex since they were copied by the same hand. However, earlier attribution to this scribe is not substantiated, as Tamani himself claimed, see G. Tamani, *Copisti e collezionisti di libri ebraici nel ducato estense*, in E. Fregni, M. Perani (eds.), *Vita e cultura ebraica nello stato estense; atti del 1 convegno internationale di studi, Nonantola, maggio 1992*, Commune di Nonantola, Nonantola 1993, p. 150. In a recent personal communication, Tamani informed me that he had no access to his notes and thus, he could not recall why he had matched

ben Solomon), a poetic composition against the pursuers of the author (or perhaps of one of his friends), who barred him from becoming a ritual slaughterer;[17] (ii) Fols. 8r-84v and 222r-222v: The anonymous and untitled arithmetical text at the heart of this article, whose contents can be summarized as follows: (a) Multiplication of integers (fols. 8r-17v), with a discussion of the nature of 1 (i.e., its non-numberness, fol. 9v), the decimal system, zero, and the various types of numbers (fols. 11v-12r); (b) Addition of integers (fols. 18r-19v); (c) Subtraction of integers (fols. 20r-22r); (d) Division of integers (fols. 22v-27v); (e) Arithmetical sequences (fols. 27v-29v); (f) Geometrical sequences (fols. 29v-31v); (g) The rule of three (fols. 32r-33r); (h) Fractions (fols. 33r-39r); (i) A wide array of verbal problems (fols. 39r-84v & 222r-222v), such as unequal division of money, distribution of wine, joint purchase of a horse, exchange of different currencies and various business transactions, as well as numerical deduction exercises based on number theory. The taught subjects are occasionally presented in larger, square, letters, as can be observed in Figures 1-4. On fol. 84v we find the following closing note. However, some arithmetical materials were added on fol. 222r-v:[18]

It is the end of [the teaching of] the methods [of calculation]	תמו אלו הדרכים
May the dweller of the upper [spheres] be blessed	ברוך שוכן
	עליונים
Amen	אמן
The writer is strong and the reader is courageous[19]	חזק הכותב ואמיץ הקורא

(iii) Fols. 85r-221v: *'Even Boḥan* ("Touchstone"), a satirical literary work composed in 1322 by the Provençal polymath Qalonymos ben Qalonymos ben Meir ha-Nasi (1287-after 1328), known as *Maestro Calo* in the Latin world. He was a poet, a philosopher, and a translator of medical, philosophical, and mathematical treatises.[20] He worked at the service of King Rob-

between Codex 33 and Isaac ben Joel of Cento, and that perhaps it was only due to the place and time of the composition. Isaac ben Joel of Cento is known to have copied a Maḥzor of Roman rite in 1397 (in elegant script, The British Library, Ms. Add. 26998). Could it have been the same scribe who copied the arithmetical treatise and the two other treatises in Ms. 33 from The Civic Library of Verona more than sixty years later? A priori, not completely impossible, but not highly probable either.

[17] I have not been able to find any information about this Netan'el ben Solomon.

[18] The heading on fol. 8r reads: בעזרת ה׳ נעשה ונצליח עזרי מעם ה׳ עושה שמים וארץ בהנ״ו עמ״י עש״ו i.e. With the help of God we shall act (literally, 'make') and we shall succeed. My help stems from God, who made heaven and earth [Psalms 121:2]. The arithmetical treatise, together with the other two texts in the codex, were copied in Cento, but we do not know where and when the *urtext* was composed and I have not found other extant copies thereof. The probability that the scribe of the codex, be it Isaac ben Joel of Cento, mentioned in footnote 16, or someone else, was actually the author of the arithmetical treatise, is rather low, because one would have expected him to identify himself as the author, especially since the names of the authors of the other two treatises are given.

[19] The first attestation of this formula is found in a codex from the East, written in 916: the formula later spread to all areas, in particular to Italy and Byzantium, see M. Beit-Arié, *Hebrew Codicology Historical and Comparative Typology of Medieval Hebrew Codices based on the Documentation of the Extant Dated Manuscripts until 1540 Using a Quantitative Approach Preprint* (internet version 0.12, 2020), p. 154 [in Hebrew].

[20] For a general outline of Qalonymos's life and

ert d'Anjou of Naples, possibly in collaboration with Judah Romano.[21] *'Even Boḥan* sheds light on Jewish life in medieval Aragon, Provence, and Italy. It also includes a poem in which the author laments having been born a man, instead of a woman. *'Even Boḥan* was first published in Naples in 1489 and has enjoyed great popularity throughout the centuries.[22]

The end of the copy of *'Even Boḥan* is marked by the following colophon (fol. 221v):

[This work] has ended and has been completed	תם ונשלם
Praise upon God	תהלה לאל
of the Universe	עולם
He giveth power to the faint.[23] The year	ברוך הנותן ליעף כח שנת
[5]221 here in Cento	ר׳כ׳א׳ פה צינטו
all night unto the morning.[24]	כל הלילה עד הבוקר

We learn that the treatises were copied in Cento in 221, i.e., 5221 Anno Mundi (=1460/1 CE). The colophon raises several questions. First, like many a colophon found in Hebrew treatises, it includes verse 29 from Chapter 40 in the Book of Isaiah, but unlike other colophons, here we find only its first half 'He giveth power to the faint'. The rest of the verse is missing: '…and to him that hath no might He increaseth strength'.[25] The colophon ends, instead, with the second half of Leviticus 6:2: 'all night unto the morning',[26] which connects rather well with the first half of Isaiah 40:29 i.e., God gave the scribe the strength to toil all night and complete his copy.

oeuvre, see J. Chotzner, *Kalonymos ben Kalonymos, a Thirteenth-Century Satirist*, in «The Jewish Quarterly Review» Vol. 13, No. 1 (1900), pp. 128-146, and for one of his mathematical angles, see T. Lévy, *L'histoire des nombres amiables : le témoignage des textes hébreux médiévaux*, in «Arabic sciences and philosophy» 6(1) (1996), pp. 63-87.

[21] Qalonymos ben Qalonymos describes Robert d'Anjou as 'secundus Salomon dicitur'. The tradition of regal intellectual patronage had already started in the beginning of the thirteenth century, in which Robert's grandfather, Charles I d'Anjou, was also involved. Working for the king, according to Immanuel the Roman, provided numerous privileges not only to Qalonymos ben Qalonymos himself but to his fellow Jews as well, see J. Shatzmiller, *Au service de la Cour de Naples: Kalonymos d'Arles et Judah Romano*, in Danièle Iancu-Agou, Carol Iancu (eds.), *L'écriture de l'histoire juive: mélanges en l'honneur de Gérard Nahon*, Paris-Louvain 2012, pp. 163-169.

[22] The *editio princeps* dates from 25 August 1489 (Naples: Yom Tov Zarfati ben Perez for Joseph ben Jacob Ashkenazi Gunzenhauser). For a list of all existing editions and translations as well as further information about the author, the treatise, and the four-century-long confusion regarding the date of its composition, see T. Dunkelgrün, *Dating the Even Bohan of Qalonymos ben Qalonymos of Arles. A microhistory of scholarship*, in «European Journal of Jewish Studies» 7:1 (2013), pp. 39-72.

[23] Isaiah 40:29. At this point, there is a change of ink by the scribe. The rest of the colophon is written in a paler and finer ink, which was also used on the next (and last) folio of the codex.

[24] Leviticus 6:2.

[25] In *Sfardata*, Malachi Beit-Arié also points to the problematicity of the colophon due to the absence of the second part of the verse.

[26] *Cf.* Judges 19:25.

The codex is eclectic. However, scientific treatises are usually found alongside other scientific works (e.g., mathematical together with astronomical ones), so one can wonder why three works of such different nature were copied together. Was the scribe creating a copy for his own library and for his personal study - the numerous erasures in the manuscript seem to indicate that it was not a commissioned work - or was the codex meant to educate a circle of Jewish students in Cento in various domains?

Judeo-Italian presence and influence in the arithmetical treatise

(i) A direct influence of the vernacular is manifest in the presence of Judeo-Italian mathematical terms. It is not uncommon to encounter Hebrew mathematical terms together with their vernacular translation, e.g., in the case of the four rudimentary arithmetical operations: multiplication, addition, subtraction, and division. Interestingly, in Ms. 33 the Judeo-Italian translation is given for the last three arithmetical operations, but not for multiplication:

The method of adding numbers, which is called *aggregare* or *ajjunjjere* or *raccoljjere*.[27]	דרך קבוץ חשבונות הנקרא אגריגארה או אייוניירי או רקוליירי

The additional meaning of the term חשבון, 'number', besides the traditional 'calculation', had been introduced by Abraham Ibn Ezra in his arithmetical treatise *Sefer ha-Mispar* ("The Book of the Number") in the twelfth century.[28]

The method of subtracting a number from a number, which is called *sottrajjere*.[29]	דרך הצאת המספר ממספר נקרא סטראייירי

The method of division of a number by a number, which is called *dividere* or *partire*.[30]	דרך חלוק מספר על מספר הנקרא דבידירי או פרטירי

An interesting lexical example is רסקולו, present with no Hebrew equivalent. רסקולו designates a line above an integer, indicating the corresponding unit fraction of the number below, e.g. 3 with רסקולו becomes 1/3, 6 becomes 1/6 etc.[31] It is perhaps connected to *risegolo* (dent), which is phonetically close (since /g/ and /k/ constitute a phonetical pair, voiced and unvoiced, respectively) or perhaps it is a diminutive form of *riso* (rice), both potentially designating the shape of the sign. Another possibility is that רסקולו is related to the verb *resecare* (to fraction).

(ii) An indirect influence of the vernacular can be discerned in some Hebrew words or formulations. As mentioned, there is no given Judeo-Italian term for multiplication, as is the case for the other three basic arithmetical

[27] See Figure 1.

[28] More on the term חשבון, see G.B.A. Sarfatti, *Mathematical terminology in Hebrew scientific literature of the Middle Ages*, Magnes Press, Jerusalem 1968 [in Hebrew], pp. 133-4. Although our anonymous treatise is not a translation of *Sefer ha-Mispar*, it was clearly influenced by it, language- and content-wise.

[29] See Figure 2.

[30] See Figure 3.

[31] Several occurrences of this term can be seen in Figure 4.

operations. However, the Hebrew verb employed here, לרבות, is not part and parcel of the traditional medieval Hebrew mathematical terminology. The common terms were either להכות (a calque from the Arabic ضرب, 'to hit', whose semantic field was extended to mean 'multiply', too) or לכפול. לרבות could be a calque of the vernacular term *moltiplicare*/*multiplicare*, which derives from the Latin *multiplicāre*, *multus* ('much' or 'many'), the same meaning of the root of the Hebrew term.[32]

Another interesting example concerns the following formulation at the beginning of a verbal problem, which does not seem to make much sense in Hebrew but rather, be the result of the author thinking in the vernacular:

Four rivers run into one spring…	ארבעה נהרות רצים אל מעיין אחד..

It takes the first 'river' 1 day to fill the spring, the second 2 days, the third 3 days, and the fourth 4 days. The reader is asked to calculate how long it would take the spring to fill up, if all 'rivers' were flowing into it simultaneously. This type of questions belongs to the medieval arithmetical tradition, which is even echoed in problems we learn to solve in school today, questions involving taps filling bathtubs or swimming pools.

We shall now focus on the two lexical difficulties in this phrase and explain how they may reflect vernacular influence. The first lexical difficulty lies in the choice of the word נהרות (rivers), which do not usually move in the direction of a spring and fill it up. The author seems to have used the wrong Hebrew word, but not in an arbitrary way. The Italian term for 'river' is *fiume*, which means both 'river' as well as 'stream'. The author probably did not realize that the semantic field of נהר does not include the meaning 'stream', as is the case in *fiume* and thus, inadvertently, created a slightly strange phrase. The second lexical difficulty in the phrase has to do with the choice of the verb. The occurrence of the verb לרוץ ('to run') in this aquatic context cannot be found in any previously known layer of pure Hebrew. Yet again, there is good reason to suspect that the author was thinking in the vernacular. The Italian verb used to describe flow (of water, for example) is *correre*[33] which means both 'to run' and 'to flow', whereas לרוץ only means 'to run'. The author, again, probably did not notice this semantic discrepancy.

Conclusions and further thoughts

As we have seen, Judeo-Italian is explicitly present in our arithmetical text in the translation of a given Hebrew mathematical term (e.g., פרטירי and דיבידירי alongside חלוק) or on its own i.e., without any Hebrew equivalent (רסקולו). At times, the influence of Judeo-Italian is implicit, such as in the case of a Hebrew term which is not part and parcel of the medieval Hebrew mathematical language but rather, seems to be a calque from the vernacular (e.g., לרבות). Further implicit influence of Judeo-Italian can be detected in Hebrew formulations which sound unnatural in Hebrew, but which would make perfect sense in the vernacular (ארבעה נהרות רצים אל מעיין אחד).

The vernacular presence in the terminology seems to serve an ancillary role, rendering the text more comprehensible for readers who may not have been familiar with the Hebrew mathematical lexicon. On the other hand, many of the more advanced arithmetical Hebrew terms were not translated into Judeo-Italian at all. Why?

[32] On the traditional medieval Hebrew mathematical bookshelf, the closest term related to the same root is רבוי, i.e., 'multitude', see Abraham bar Ḥiyya's (or Ḥayya's) rendition of Euclid's definition of the number (Book VII, Definition 2): המספר הוא הרבוי הנקבץ מן האחדים i.e. 'A number is a multitude composed of units', see M. Guttman (ed.), *Ḥibbur ha-meshiḥa veha-tishboret*, Berlin, 1913, p. 11.

[33] Alongside 'scorrere', see https://www.treccani.it/vocabolario/correre/

Did the author not know their Italian equivalent, or did he not think that his readers would recognize the Italian terms and thus, their translation would not be of help? Is there some randomness in the process of choosing which terms are translated into Judeo-Italian and which ones are not? A better understanding of this phenomenon could perhaps be obtained by detecting and analysing more such texts.[34]

One must not forget that the vernacular was the native and daily language of Jews on the Italian Peninsula, and in contrast to other Jewish authors (in particular, Sephardic ones), who preserved a pure Hebrew vocabulary in their scientific writings, Italian Jewish scholars, albeit being capable of reading and writing in Hebrew, often did not stick to purism when writing in Hebrew. As we have seen, they were, also, at times, inadvertently thinking in the vernacular while writing in Hebrew, unaware of the extent of the semantic field of Hebrew general notions (לרוץ, נהר) in relation to that of the Italian words they were translating (*correre*, *fiume*).

Finally, it is important to note that apart from one or more potential Hebrew mathematical sources of our arithmetical text (one immediate suspect being Abraham Ibn Ezra's *Sefer ha-Mispar*), part of it could be derived from one or several *abbaco* vernacular sources (hitherto unidentified), or perhaps, from arithmetic lessons given to the author by a Christian master. The Hebrew treatise shares common features with vernacular *abbaco* books of the time, in terms of its arithmetical contents and structure. Concomitantly, one must not forget that this branch of mathematics had already made its way into the medieval Hebrew mathematical bookshelf through the Arabic tradition of practical arithmetic, *Ḥisāb* (in which *abbaco* finds its roots, too). And yet, given the numerous vernacular terms in our Hebrew text, it is unlikely that our Hebrew treatise is completely detached from the vernacular arithmetical context.

A full mapping and analysis of all the mathematical terms in our text (both Hebrew and the vernacular) and their possible connection with earlier Hebrew and perhaps vernacular arithmetical sources will be of great interest. Not only will it increase our understanding of the evolution of Hebrew as a language of science in the Judeo-Italian linguistic environment, but it could also shed light on the contact of the Jewish scholars with mathematical books and masters in their Christian environment. Furthermore, the investigation of how the vernacular influenced the Hebrew formulations, be they mathematical or not, is very important for understanding how the daily language of Italian Jews influenced their written Hebrew.

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SUMMARY

My article presents a preliminary study of the impact of the vernacular in an anonymous Hebrew arithmetical treatise copied in Cento in 1460/1. After a short contextual introduction about Jews in Cento, Judeo-Italian, and the *abbaco* tradition, I discuss the codex in which the mathematical composition is found (Verona, Biblioteca Civica Ms. 33) and I analyse several examples of textual manifestations of the vernacular within.

KEYWORDS: Arithmetic; Hebrew; Vernacular.

[34] I am currently researching the vernacular (as well as Arabic) linguistic influence within Berachia ben Jacob Gallico's Hebrew arithmetical treatise *Mevo' ha-Ne'arim* (Gateway for the Youths).

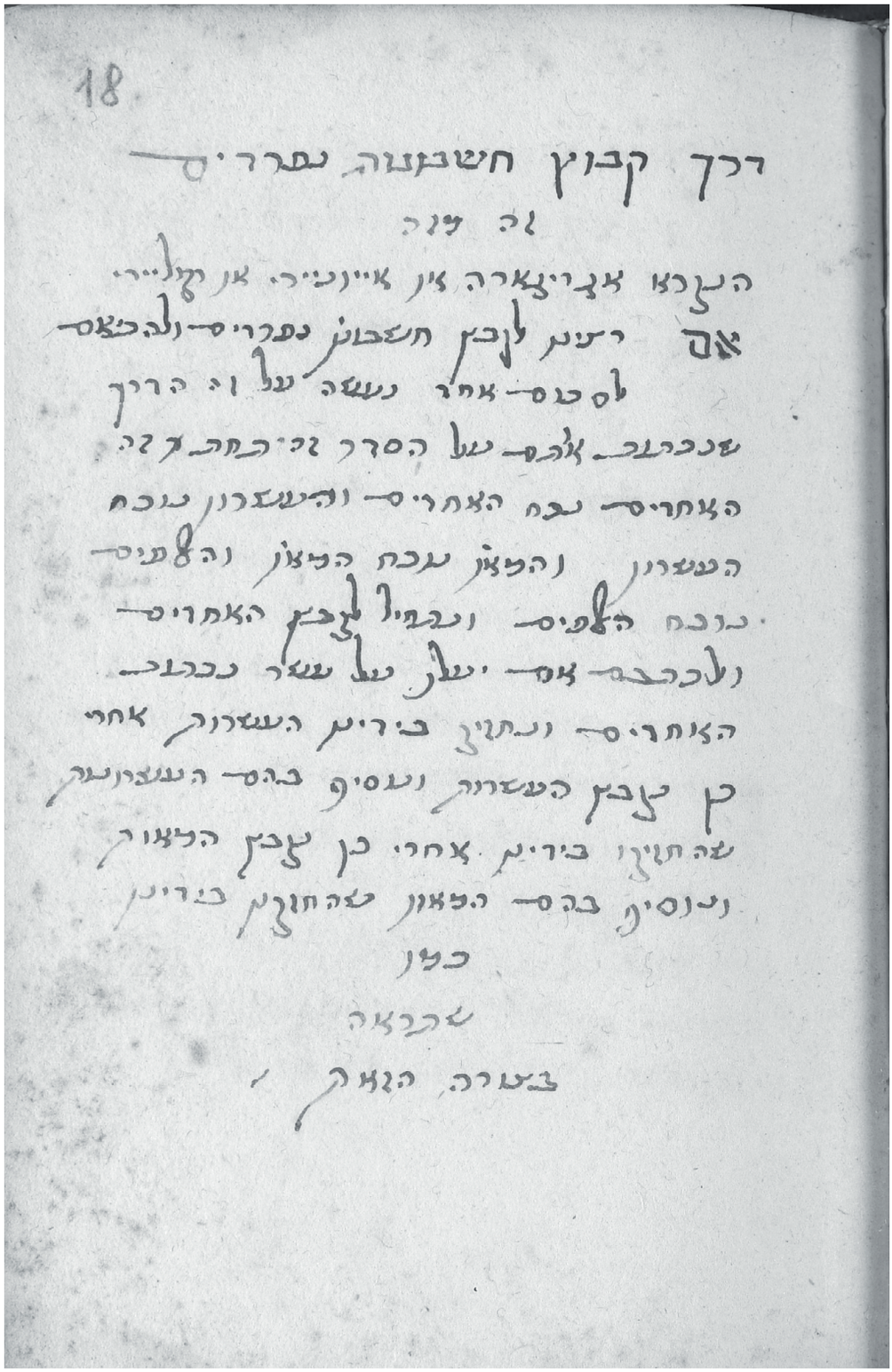

Figure 1 - The beginning of the part on addition.
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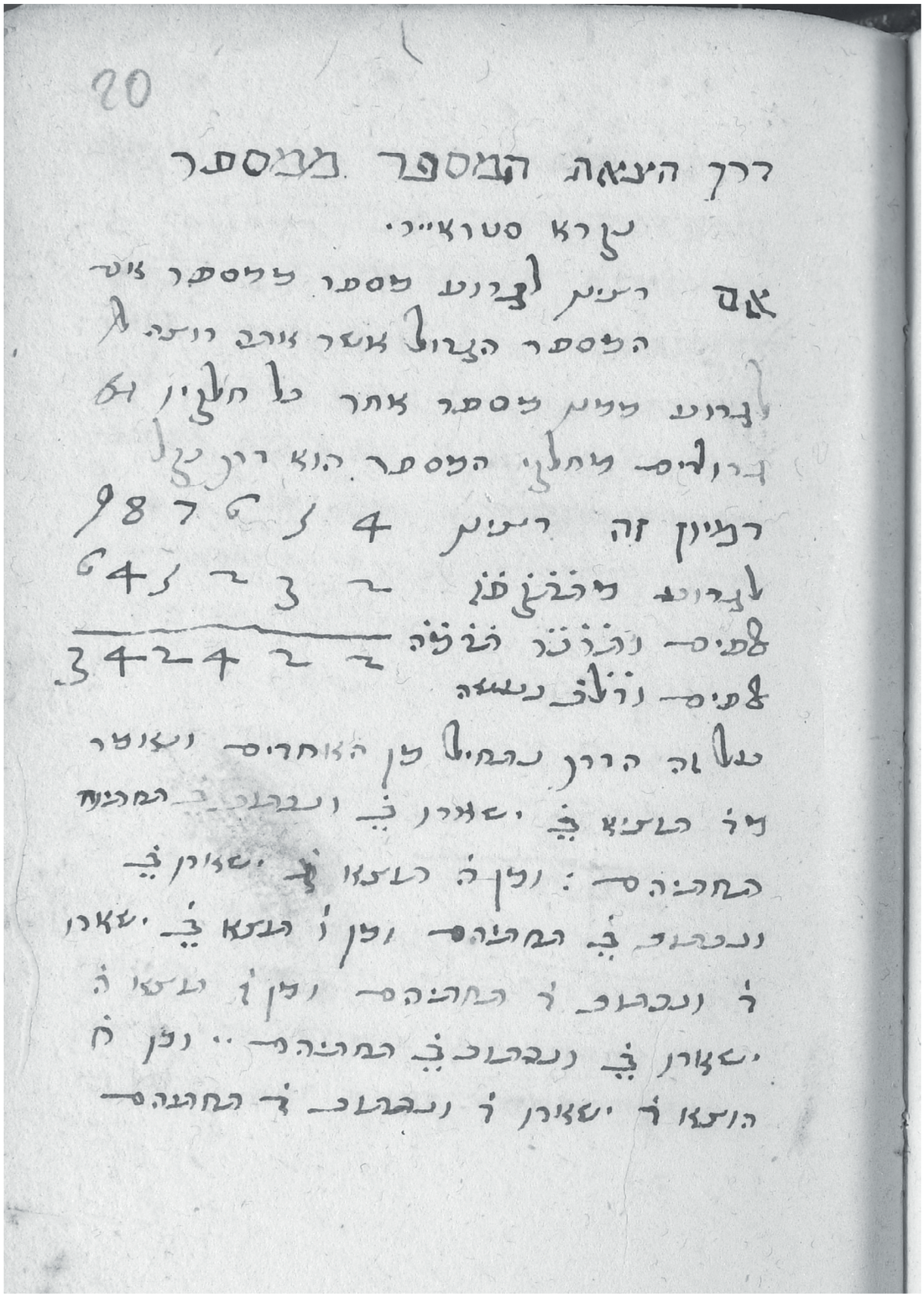

Figure 2 - The beginning of the part on subtraction.
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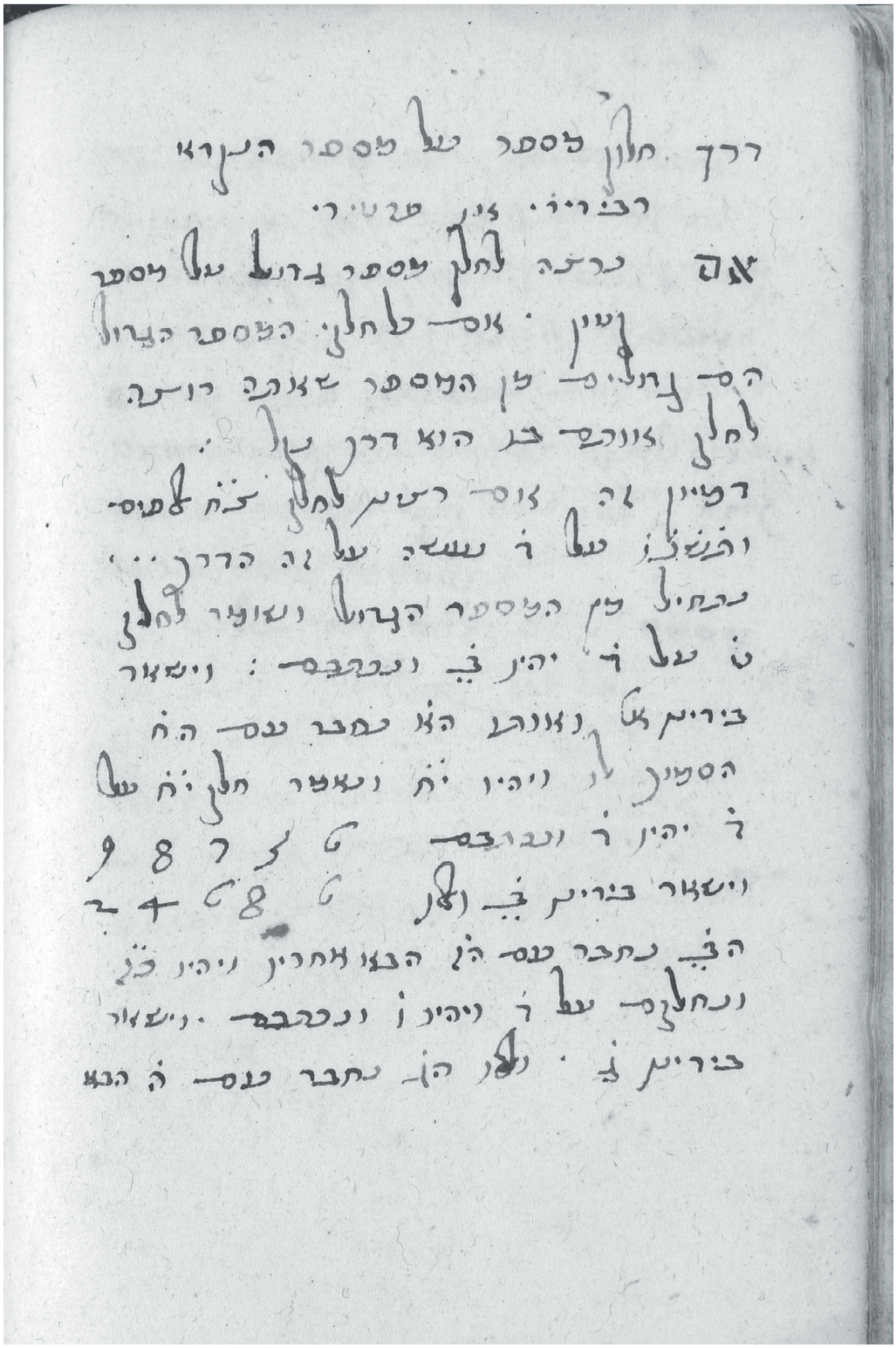

Figure 3 - The beginning of the part on division.
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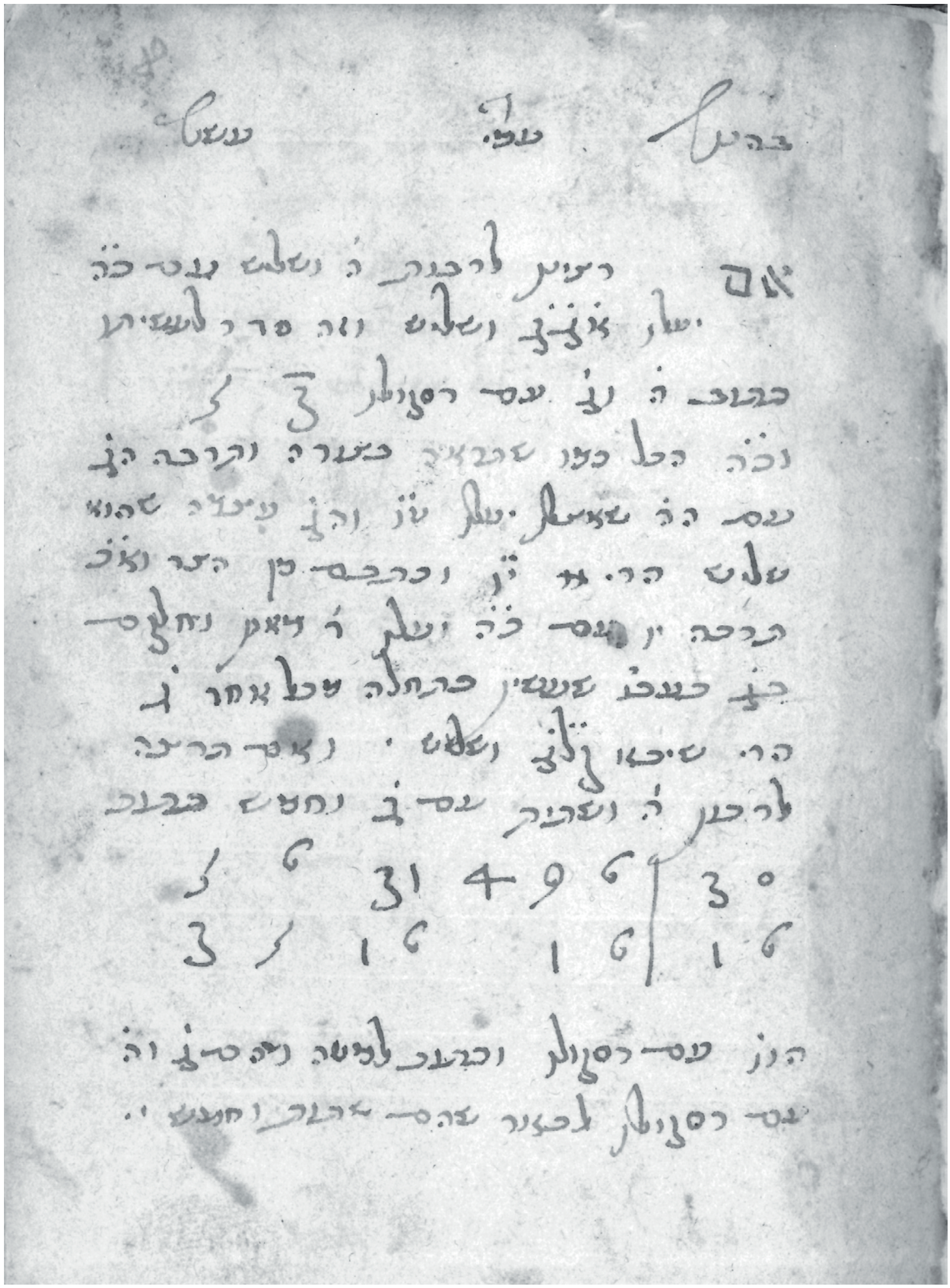

Figure 4 - The beginning of the part on multiplication.
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